Validating **time-of-detection** methods for alerting an upcoming critical transition: a trade-off between sensitivity and specificity.

Monitoring early-warning signals (EWS) in infectious disease data could be used to inform ***when*** a disease went through elimination

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What are early-warning signals?

EWS are time-changing statistics which behave in a predictable way on the approach towards a critical transition. Critical Slowing Down phenomenon manifests itself in **increased autocorrelation, variance and magnitude of fluctuations** as a system approaches a transition, due to the system's slow recovery from perturbations as its dominant eigenvalue approaches zero[1].

EWS are updated in **real-time**, can be **automated** and are **computationally efficient.**

This is key in infectious disease modelling to assess when the basic reproduction number (R_0) has reduced below the threshold of one[2].

What is the time-of-detection?

Time-of-detection is the first time when there is significant evidence of an impeding critical transition.

(a) Is the time of first detection prior to the time the bifurcation occurs?

(a) How long is the lead-time period between detecting and reaching the bifurcation?

(a) What is the TPR and FPR of this prediction?

References

[1] Scheffer, M, et al. "Early-warning signals for critical transitions." Nature (2009)

[2] Southall, E, et al. "Prospects for detecting early warning signals in discrete event sequence data: Application to epidemiological incidence data." PLoS Comp Bio (2020)

[3] Drake, J & Griffen, B. "Early warning signals of extinction in deteriorating environments." Nature (2010)

[4] Shiryaev, A "Quickest detection problems: Fifty years later." Sequential Analysis (2010)

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Validation Methods

We validate an empirical study which offered lead time predictions using EWS: **normalised composite**[3], and compared the performance to a method in change-point analysis from statistics called **Quickest Detection**[4].

For each method we present:

STEP 1 (timeseries)
A demonstration for simulated data which is:

- Going through bifurcation (disease elimination) in green. We **want to** trigger a bifurcation
- At steady state in blue. We **do not** want to trigger a bifurcation.

The first "time-of-detection" is highlighted with a red star. The bifurcation point is the vertical dotted line.

STEP 2 (heatmaps)
An extension of each method by varying the number of consecutive points required to exceed the threshold for a detection to be triggered.

From 500 simulations undergoing a bifurcation (disease elimination),

$$TPR = \frac{\text{bifurcating simulations which detect a bifurcation}}{500}.$$

From 500 simulations at steady state,

$$FPR = \frac{\text{steady state simulations which detect a bifurcation}}{500}$$

We plot the power metric

$$\text{Power Metric} = TPR - FPR, \text{Power Metric} \in [-1, 1],$$

A score close to 1 (coloured red) indicates a **high sensitivity and high specificity**. A score close to −1 (coloured blue) indicates a **low sensitivity and low specificity**.

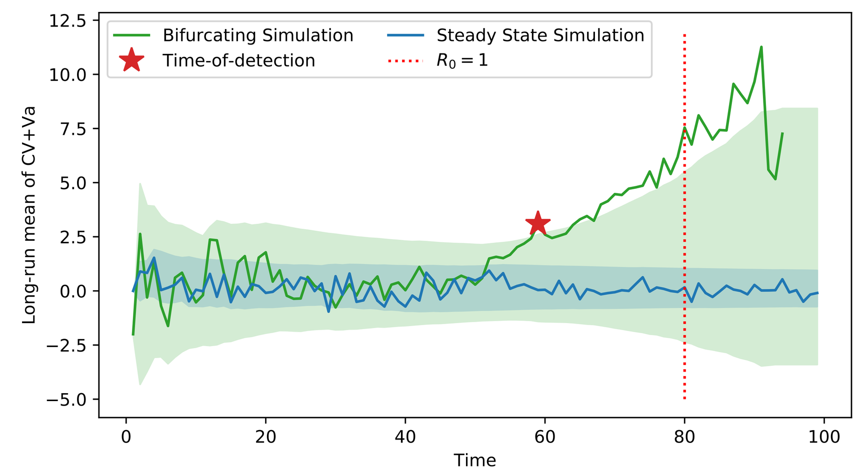
STEP 3 (boxplots)
For the best EWS and number of consecutive points (chosen from STEP 2 heatmap), we calculate the time-of-detection for each simulation.
Each boxplot is the distribution of time-of-detection from each simulation (each simulation data point is shown with grey dots), where:

- **Green Boxplots:** simulations going through a bifurcation. Want to return time-of-detection for all 500 simulations (high number of dots). *TPR* given in legend.
- **Blue Boxplots:** simulations at steady state. DO NOT want to return time-of-detection for any simulations (low number of dots). FPR_1 given in legend.
- **Purple Boxplots:** simulations where incidence is reducing but the system **does not** undergo a bifurcation. DO NOT want to return time-of-detection for any simulations (low number of dots). FPR_2 given in legend.

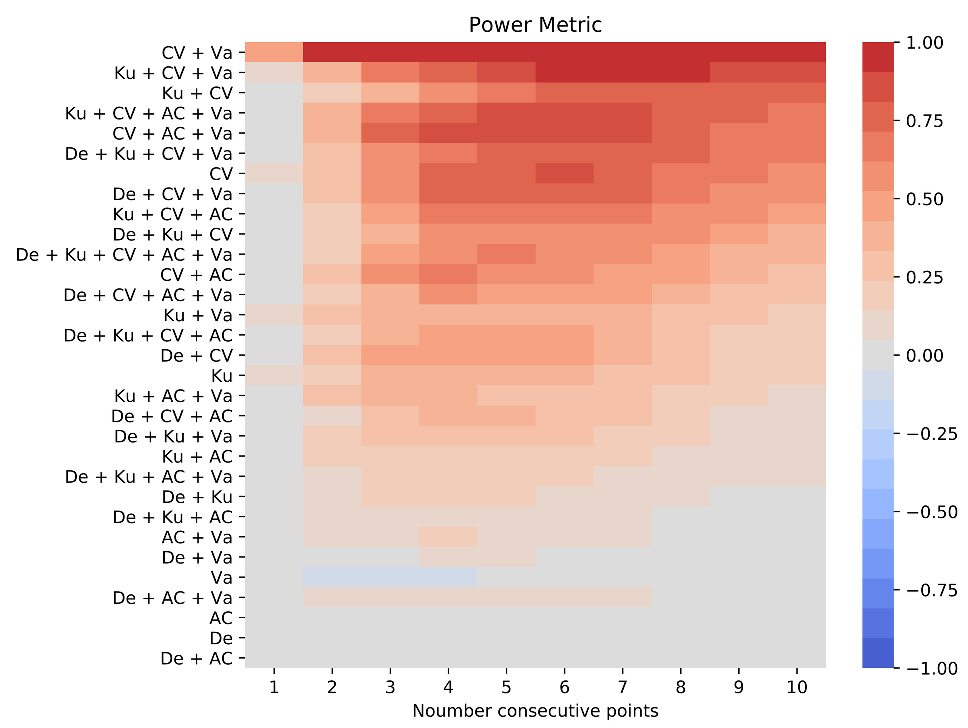
Normalised Composite Method[3]

The **normalised composite** of multiple EWS is calculated. If the composite exceeds the long-run mean plus two standard-deviations, a detection is triggered.

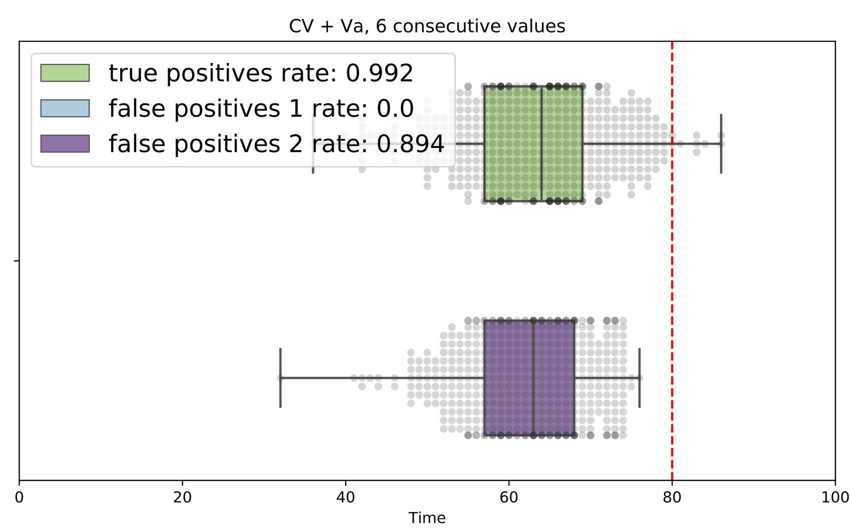
STEP 1: Demonstration using the composition of variance and coefficient of variation.



STEP 2:



STEP 3:



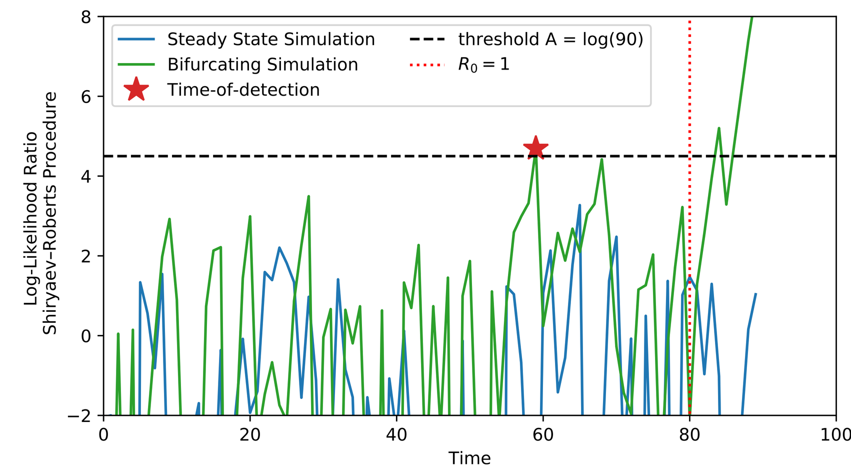
Conclusions:

- CV + Va is highly specific and sensitive for all consecutive values
- Indicators more specific when at least 5 consecutive points are considered
- AC, De, Va (and their combinations) are poor EWS
- High false detection rate on "changing-not-bifurcating" data (purple)

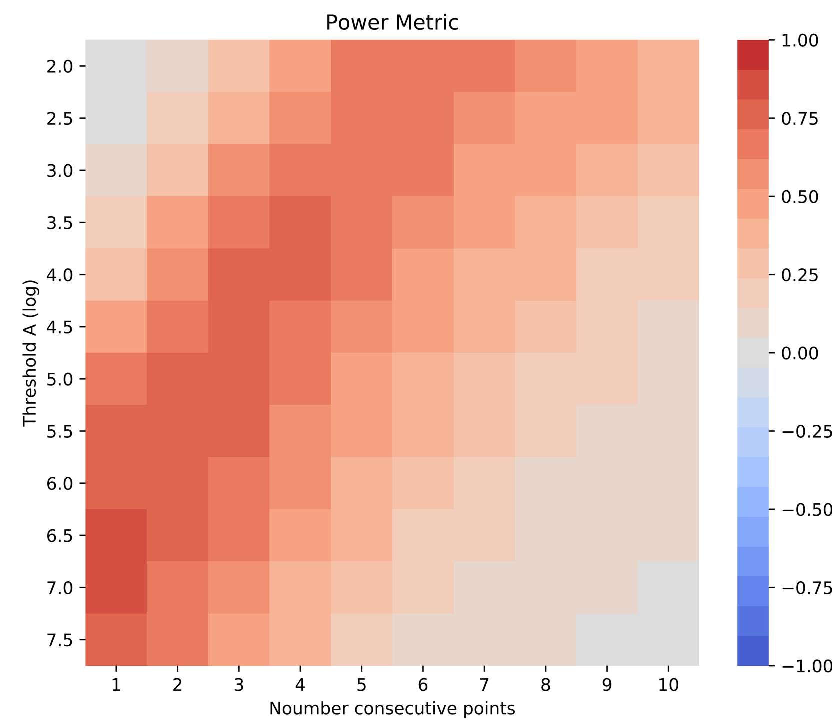
Quickest Detection Method[4]

The **Quickest Detection** method from change-point analysis employs two probability densities describing the data pre- vs post-bifurcation. A detection is triggered when the Shiryaev–Roberts statistic (based of the likelihood ratio) exceeds a threshold A.

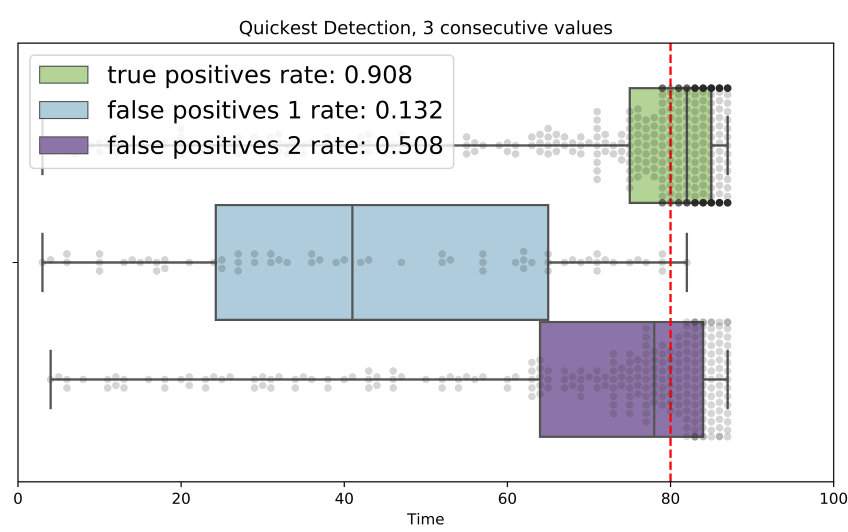
STEP 1: Demonstration using $x_i \in \mathcal{N}(0, 30), i \in [0, \tau]$ and $x_i \in \mathcal{N}(0, 10), i \in [\tau + 1, T]$, with $log(A) = T$.



STEP 2:



STEP 3:



Conclusions:

- Requires the user to define the probability distributions and threshold
- Small to no lead-time
- Best method for data which is "changing-not-bifurcating" (purple)